



NS – 309

V Semester B.A./B.Sc. Examination, Nov./Dec. 2016  
(Semester Scheme)  
(Fresh) (CBCS) (2016-17 Onwards)  
MATHEMATICS – V

Time : 3 Hours

Max. Marks : 70

*Instruction : Answer all questions.*

PART – A

Answer any five questions :

(5×2=10)

1. a) In a ring  $(R, +, \cdot)$ , prove that  $(-a) \cdot (-b) = a \cdot b; \forall a, b \in R$ .
- b) Define subring of a ring. Give an example.
- c) Give an example of
  - i) Commutative ring without unity
  - ii) Non-commutative ring with unity.
- d) Find the unit vector normal to the surface  $xy^2z^2 = 4$  at  $(-1, -1, 2)$ .
- e) Find the divergence of  $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ .
- f) Prove that  $E = e^{hD}$ .
- g) Write Lagranges Interpolation Formula.
- h) Evaluate  $\int_0^1 \frac{dx}{1+x}$  by Simpson's  $\frac{3}{8}$  rule.



where

<b>x</b>	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
<b>y = f (x)</b>	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.



## PART - B

Answer **two full** questions.

(2×10=20)

2. a) Show that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are

- i)  $a \in S, b \in S \Rightarrow a - b \in S$
- ii)  $a \in S, b \in S \Rightarrow ab \in S$ .

b) Prove that every field is an Integral Domain.

OR

3. a) Show that the set of all matrices of the form  $\left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} / a, b \in R \right\}$  is a non-commutative ring without unity with respect to addition and multiplication of matrices.

b) Fill all the principal ideals of a ring  $R = \{0, 1, 2, 3, 4, 5\}$  w.r.t.  $+_6$  and  $\times_6$ .

4. a) Prove that  $(\mathbb{Z}_7, +_7, \times_7)$  is a commutative ring with unity. Is it a Integral Domain ?

b) State and prove fundamental theorem of homomorphism.

OR

5. a) Prove that a commutative ring with unity is a field if it has no proper ideals.

b) Prove that the mapping  $f: (Z, +, \times) \rightarrow (2Z, +, *)$  where  $a * b = \frac{ab}{2}$  defined by  $f(x) = 2x, \forall x \in Z$  is an isomorphism.

## PART - C

Answer **two full** questions.

(2×10=20)

6. a) Find the directional derivative of  $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$  at the point  $(1, 1, -1)$  in the direction of  $2\hat{i} - \hat{j} + \hat{k}$ .

b) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $r = |\vec{r}|$ , prove that

$$\text{i) } \nabla r^n = n r^{n-2} \vec{r} \qquad \text{ii) } \nabla \left( \frac{1}{r} \right) = \frac{-\vec{r}}{r^3}.$$

OR



7. a) Show that the surfaces  $4x^2y + z^3 = 4$  and  $5x^2y - 2yz = 9x$  intersect orthogonally at the point  $(1, -1, 2)$ .

b) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , show that  $\nabla^2 \left( \text{div} \left( \frac{\vec{r}}{r^2} \right) \right) = \frac{2}{r^4}$ .

8. a) If  $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$ , find  $\text{div} \vec{F}$  and  $\text{curl} \vec{F}$ .

b) If  $\phi$  is scalar point function and  $\vec{F}$  is vector point function then  $\text{curl} (\phi \vec{F}) = \phi \text{curl} \vec{F} + (\text{grad} \phi) \times \vec{F}$ .

OR

9. a) If  $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ , find a, b, c such that  $\vec{F}$  is irrotational then find  $\phi$  such that  $\vec{F} = \nabla\phi$ .

b) Prove that  $\text{curl} (\text{curl} \vec{f}) = \text{grad} (\text{div} \vec{f}) - \nabla^2 \vec{f}$ .

PART - D

Answer two full questions.

(2x10=20)

10. a) Find a cubic polynomial which takes the following data :

x	0	1	2	3
f(x)	1	2	1	10

b) Find f(1.4) from the following data.

x	1	2	3	4	5
f(x)	1	8	27	64	125

using difference table.

OR

11. a) Evaluate  $\Delta(e^{3x} \log 4x)$ .

b) Find f(7.5) from the following data.

x	7	8	9	10
f(x)	3	1	1	9

using difference table.





12. a) Using Newton's divided difference formula find  $f(3)$  from the given data.

<b>x</b>	0	1	2	5
<b>f(x)</b>	2	3	12	147

- b) Evaluate  $\int_0^8 \frac{dx}{1+x^2}$  by using Simpson's  $\frac{3}{8}$  rule.

OR

13. a) Using Lagrange's interpolation formula find  $f(2)$  from the following data.

<b>x</b>	0	1	3	4
<b>f(x)</b>	5	6	50	105

- b) Using Simpson's  $\frac{1}{3}$  rule, evaluate  $\int_0^{0.6} e^{-x^2} dx$ .

